Decisiveness and
The Viability of Anarchy

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Abstract

This paper explores how the viability of anarchy depends on the decisiveness parameter that determines the marginal effect of allocating a resource to an appropriative competition. In a one-factor model in which agents use in the appropriative competition the same resource that they are competing to appropriate, anarchy appears to be fragile, because in this model equilibrium consumption would be adequate for the viability of anarchy only if the decisiveness parameter were small. But, we show that this analysis is not robust. Specifically, we find that in a more credible model, in which the resource that agents compete to appropriate and the resource that agents use in the appropriative competition are distinct, anarchy is viable even if the decisiveness parameter is large.

Anarchy is an unorganized social system in which autonomous agents compete to appropriate resources.[1] Nevertheless, Jack Hirshleifer (1995, page 26) tells us that, despite the absence of organization, "anarchy...is not chaos, but rather a spontaneous order". Similarly, Helen Milner (1991, page 70) points out that scholars who view the relations among sovereign states to be anarchic find "regularized, predictable patterns of behavior among states [and] order lurking in the seeming chaos of international relations". Milner (1991, page 74) argues that anarchy differs from an organized social system like a state only in "the manner in which order is provided".

In fact, anarchy is prevalent. We observe many long-lasting anarchic equilibria, ranging from external relations among sovereign states to the environments in which much of the world's population, both urban and rural, lives.[2]

Curiously, although he gives many examples of anarchy, Hirshleifer (1995, page 26) also claims that "anarchy is fragile". The argument goes as follows: To be viable a social system must generate "adequate" consumption, defined (Hirshleifer 1995, page 33) to be the "minimum...required to sustain life for an individual actor or for a group to preserve its

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[1]This definition of anarchy accords with the definition typically used by economists. See, for example, Dan Usher(1992), Jack Hirshleifer (1995), and Herschel Grossman (2000). Hirshleifer contrasts anarchy with "amorphy", a situation in which, as in the state of nature imagined by Rousseau, people exist in isolation from each other. Hirshleifer also contrasts anarchy with organized social systems, in which category he includes both "hierarchy", a system in which a higher authority, such as a state or a super state, regulates the appropriation of resources, as well as systems of informal social controls over the appropriation of resources. In contrast, the literature that analyzes informal social controls as an alternative to a state or a super state often uses a broader definition of anarchy, according to which anarchy precludes specialized authorities who enforce collective choices, but anarchy does not preclude socially constructed punishments that support cooperative equilibria. For extended discussions of the possibilities for cooperative equilibria, see, for example, Michael Taylor (1982), Robert Axelrod (1984), Emerson Niou and Peter Ordeshook (1990), or, more recently, Abhinay Muthoo (2000).

[2]Juan Mendoza (1999) argues that in many countries, even though a state exists, anarchy prevails because the state chooses to free ride on the efforts of private agents to create and to protect property rights.

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institutional integrity". Moreover, with appropriative competition and the production of consumables being alternative uses of a resource, equilibrium consumption depends negatively on the decisiveness parameter that determines the marginal effect of allocating the resource to the appropriative competition.

In addition, Hirshleifer assumes that agents use in the appropriative competition the same resource that they are competing to appropriate. As this assumption presents a chicken-and-egg conundrum, presumably it is intended to be a convenient simplification, not meant to be taken literally. Under this assumption, and with a large number of agents competing to appropriate a large amount of the resource, it turns out that the negative effect of the decisiveness parameter on equilibrium consumption is so strong that equilibrium consumption would be positive only if the decisiveness parameter were smaller than one. Because we have no reason to presume that the decisiveness parameter is smaller than one, let alone as much smaller than one as this model would require for equilibrium consumption to be adequate, this analysis suggests that anarchy is unlikely to be viable.[3]

But, is this suggestion that anarchy is fragile robust? Does theory belie the observation that anarchy is prevalent? In this paper we examine the viability of anarchy in a more general model that allows the resource that agents compete to appropriate and the resource that agents use in the appropriative competition to be distinct. This model avoids the chicken-and-egg conundrum associated with Hirshleifer's one-factor assumption. More importantly, our analysis reveals that with the more credible assumption of two distinct factors, as long as adequate consumption is not too large, anarchy is viable even if the decisiveness parameter is much larger than one.[4]

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[3]What happens if the decisiveness parameter is too large for anarchy to be viable? According to Hirshleifer (1995, page 48), "Anarchy is always liable to 'break down' into amorphy or 'break up' into organization."

[4]Of course, anarchy can fail to survive for other reasons. Most obviously, because a state or a super state can enforce collective choices, forming a state or a super state can be an attractive alternative to anarchy. For more on these issues, see Grossman (2000).

1. A Generic Model of Anarchy

Consider an anarchic situation in which a group of $n+1$ identical autonomous agents, $n \in \{1, 2, 3, ...\}$, appropriate among themselves $(n+1)E$ units of a heretofore unappropriated resource.[5] We assume that this resource is an input into the production of consumables, but the analysis would be isomorphic if the resource itself were consumable. The agents can be individuals, or they can be groups, such as families or tribes, who act as unitary agents. The agents can even be sovereign states, again as long as we can assume that they act as unitary agents.

To model the appropriative competition, assume that, as in a standard "contest-success function", an agent appropriates e units of the heretofore unappropriated resource, where

$$(1) \qquad e = \frac{r^m}{r^m + nR^m}(n+1)E, \qquad m > 0.$$

In equation (1), r is the amount of a resource that an agent allocates to the appropriative competition, and R is the amount of that resource that other agents on average allocate to the appropriative competition. The parameter m is the "decisiveness parameter".[6]

We assume that the resource allocated to appropriative competition has an alternative use as an input into the production of consumables. But, for now we do not specify whether

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[5]For simplicity, we follow most of the literature in abstracting from initial claims to this resource. With initial claims, as in Grossman and Minseong Kim (1995) and in Grossman (2001), we would have to allow for the distinction between the challenging of initial claims and the defending of initial claims.

[6]Equation (1) is a black box that does not explicitly specify the process of appropriation, just as the standard generic production function does not explicitly specify the process of production. In other words, equation (1) does not tell us how agents appropriate the heretofore unappropriated resource any more than a production function tells us how to make cars. The relation between the amounts allocated to the appropriative competition and the appropriative outcome described by equation (1) could involve such disparate processes as a nonviolent scramble, a division under the threat of force, or a violent struggle. We could easily extend the analysis to allow the appropriative competition to despoil some of the heretofore unappropriated resource by assuming that E is a decreasing function of R.

and, if so, how the heretofore unappropriated resource differs from the resource that agents allocate to the appropriative competition.

Equation (1) exhibits several important properties that we can plausibly consider to be generic to the appropriation of a heretofore unappropriated resource. First, e is bounded between zero and $(n+1)E$, the entire amount of the heretofore unappropriated resource. Second, $\partial e/\partial r$ is positive. Third, if r equals R, then e equals E.

We can see from equation (1) why the decisiveness parameter is important. A straightforward calculation reveals that $\partial^2 e/\partial r \partial m$, evaluated at r equal to R, is positive.[7] This property means that in a symmetrical equilibrium, in which every person makes the same choice of r, the larger is the decisiveness parameter the larger is the marginal effect of allocating more to the appropriative competition on the amount of the heretofore unappropriated resource that an agent appropriates.

An agent's consumption, denoted by c, depends positively on the amount of the heretofore unappropriated resource that the agent appropriates and depends negatively on the amount that the agent allocates to the appropriative competition. Specifically, we assume a production technology such that

$$(2) \qquad\qquad\qquad c = F(e,r),$$

where $\partial F(e,r)/\partial e > 0$, $\partial^2 F(e,r)/\partial e^2 \leq 0$, $\partial F(e,r)/\partial r < 0$, and $\partial^2 F(e,r)/\partial r^2 \leq 0$. For now we do not specify any other properties of the function $F(e,r)$. Let $\underline{c}$ denote the minimum level of per capita consumption that is adequate for the viability of anarchy.[8]

An agent chooses r to maximize its consumption, taking R as given. Using equations

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[7]The sign of $\partial^2 e/\partial r \partial m$ is the same as the sign of $[1 + m\ ln(r) + m\ ln(R)]\ (r^m + nR^m) - 2m\ [r^m\ ln(r) + nR^m\ ln(R)]$. With r equal to R this expression becomes $(n+1)\ R^m$.

[8]If the heretofore unappropriated resource and the resource that the agent allocates to appropriative competition were themselves consumable, then we would interpret $F(e,r)$ to be a utility function, and we would interpret $\underline{c}$ to be the minimum level of utility that is adequate for the viability of anarchy.

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(1) and (2) the first order condition for the maximization of an agent's consumption implies

$$(3) \qquad \frac{dc}{dr} = \frac{\partial F(e,r)}{\partial e}\frac{\partial e}{\partial r} + \frac{\partial F(e,r)}{\partial r} = 0,$$

$$\text{where} \quad \frac{\partial e}{\partial r} = \frac{mr^{m-1}nR^m}{(r^m + nR^m)^2}(n+1)E.$$

We assume that the second order condition, $d^2c/dr^2 < 0$, is satisfied.

In a symmetrical equilibrium in which every agent makes the same choice of r, we have for every agent $r = R$ and, hence, $e = E$. Accordingly, in a symmetrical equilibrium equation (3) becomes

$$(4) \qquad \frac{dc}{dr} = \frac{\partial F(E,R)}{\partial e}\, m\, \frac{n}{n+1}\frac{E}{R} + \frac{\partial F(E,R)}{\partial r} = 0.$$

Let R^* denote the value of R that solves equation (4). Because $\partial F(e,r)/\partial e$ is positive, given that the second-order condition is satisfied, equation (4) implies that dR^*/dm is positive. In other words, with each agent maximizing its consumption, the larger is the decisiveness parameter, m, the larger is the equilibrium allocation to the appropriative competition, R^*.

More precisely, equation (4) implies that R^* satisfies

$$(5) \qquad m\, \frac{n}{n+1}\frac{E}{R^*} = -\frac{\partial F(E,R^*)}{\partial r}\bigg/\frac{\partial F(E,R^*)}{\partial e}.$$

The left side of equation (5) is the marginal effect of allocating units of a resource to the appropriative competition on amount of the heretofore unappropriated resource that an agent appropriates. The right side of equation (5) is the ratio of the marginal productivity of the heretofore unappropriated resource to the marginal productivity of the resource that agents allocate to the appropriative competition.

Let c^* denote the equilibrium value of per capita consumption. In a symmetrical equilibrium c^* equals $F(E,R^*)$. Because $\partial F(e,r)/\partial r$ is negative and dR^*/dm is positive, dc^*/dm is negative. In other words, the larger is the decisiveness parameter, m, the smaller is the equilibrium value of per capita consumption.

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The viability of anarchy requires that c^* be at least as large as $\underline{c}$. The result that dc^*/dm is negative suggests that for large enough values of m this condition might not be satisfied. Let $\overline{m}$ denote the maximum value of m that is consistent with c^* being as large as $\underline{c}$. The viability of anarchy requires that m be not larger than $\overline{m}$. We turn next to the determination of $\overline{m}$ for specific examples of the production technology, $F(e, r)$.

Before proceeding it is useful to observe that in the limit, if R were to approach zero, then per capita consumption would approach $F(E, 0)$. Let $\hat{c}$ denote $F(E, 0)$. We call $\hat{c}$ potential consumption.

2. A One-Factor Model

Consider a simple version of this generic model in which agents use in the appropriative competition the same resource that they are competing to appropriate. As we have noted, taken literally, the use of the heretofore unappropriated resource to appropriate that resource presents a chicken-and-egg conundrum. The question is whether this assumption is an innocent simplification.

To implement this assumption, specialize equation (2) by assuming that an agent's consumption is an increasing concave function of the difference between the amount of the resource that the agent appropriates and the amount of that resource that the agent allocates to the appropriative competition. Specifically, assume that

(6) $$c = F(e, r) = (e - r)^{\gamma}, \qquad 0 < \gamma < 1.$$

Equation (6) implies that equilibrium consumption, c^*, equals $(E - R^*)^{\gamma}$. Thus, c^* is a decreasing concave function of R^*. In addition, the relation between c^* and R^* is such that, for any positive value of adequate consumption, $\underline{c}$, no matter how small, a large enough value of R^* would be inconsistent with the requirement that c^* be as large as $\underline{c}$. More precisely, for c^* to be as large as $\underline{c}$, R^* cannot be larger than $E - \underline{c}^{1/\gamma}$.

Equation (6) also implies that the ratio of $-\partial F(E, R^*)/\partial r$ to $\partial F(E, R^*)/\partial e$ equals one. Hence, the equilibrium condition for determining R^*, equation (5), requires that the

marginal effect of allocating a resource to the appropriative competition on the amount that an agent appropriates equals one. Specifically, equation (5) becomes

$$(7) \qquad R^* = m \; \frac{n}{n+1} \; E.$$

Equation (7) implies that R^* is proportionate to m. Hence, given that c^* is a decreasing concave function of R^*, equation (7) also implies that c^* is a decreasing concave function of m. In addition, equation (7) implies that c^* is positive if and only if m is smaller than $1 + 1/n$. The possibility that in equilibrium consumption would not be positive arises because in the one-factor model agents can choose to waste the entire resource endowment in appropriative competition. The concave locus in Figure 1 illustrates the relation between c^* and m for the one-factor model.

Let $\overline{m}_1$ denote the value of $\overline{m}$, the maximum value of m that is consistent with c^* being as large as $\underline{c}$, in the one-factor model. By the definition of $\overline{m}$, for m equal to $\overline{m}_1$, c^* equals $\underline{c}$. By varying $\underline{c}$ in Figure 1, we see that $\overline{m}_1$ is a decreasing concave function of $\underline{c}$. More precisely, using equation (7), the condition for the viability of anarchy, $c^* \geq \underline{c}$, becomes $m \leq \overline{m}_1$, where

$$(8) \qquad \overline{m}_1 \; = \; \left[1 - \left(\frac{\underline{c}}{\hat{c}} \right)^{1/\gamma} \right] \frac{n+1}{n}.$$

According to equation (8), as $\underline{c}$ approaches potential consumption, $\hat{c}$, where $\hat{c}$ equals E^γ, $\overline{m}_1$ approaches zero. This result implies that, if adequate consumption is almost as large as potential consumption, then anarchy is unlikely to be viable. Even with a small decisiveness parameter, and hence a small equilibrium allocation to the appropriative competition, consumption would be smaller than adequate consumption.

Equation (8) also implies that, as $\underline{c}$ approaches zero, $\overline{m}_1$ approaches $1 + 1/n$. This result says that with a large number of agents competing to appropriate a large amount of a heretofore unappropriated resource, even if adequate consumption is small, the maximum value of the decisiveness parameter that is consistent with the viability of anarchy is not larger
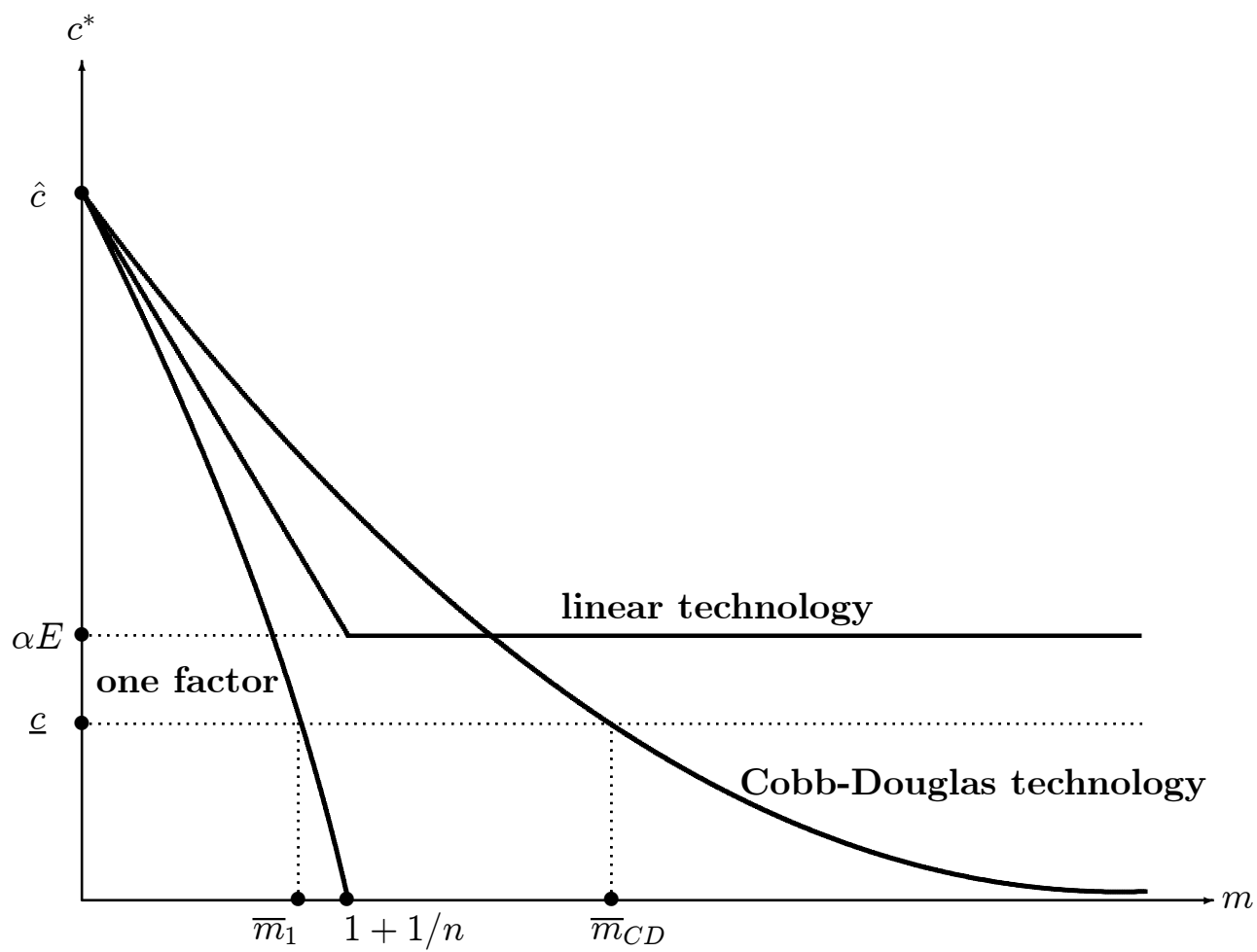
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Figure 1: Equilibrium Consumption

than one. With a small number of agents competing to appropriate a proportionately smaller amount of a heretofore unappropriated resource, the maximum decisiveness parameter that is consistent with the viability of anarchy would be larger. But, even with only two agents, the maximum decisiveness parameter would be only twice what it would be with a large number of agents.

3. Two-Factor Models

To explore the robustness of this analysis, consider an alternative setup in which the heretofore unappropriated resource and the resource used in the appropriative competition are distinct. Specifically, assume that each agent is endowed with one unit of an inalienable resource, which we can take to be time and effort. Assume further that agents use time and effort to appropriate the heretofore unappropriated resource. Thus, in equation (1) r now represents the fraction of its time and effort that an agent allocates to appropriative competition. This setup avoids the chicken-and-egg conundrum of the one-factor model.

Assume further that an agent's consumption depends on the amount that the agent appropriates and on the amount of time and effort that the agent allocates to production according to a standard constant-elasticity-of-substitution (CES) technology,

$$(9) \qquad c = F(e, r) = [\alpha e^\sigma + \beta(1 - r)^\sigma]^{1/\sigma}, \qquad 0 < \alpha < 1, \quad 0 < \beta < 1, \quad \sigma \le 1.$$

In equation (9) $1 - r$ is the fraction of its time and effort that an agent allocates to production. The constant elasticity of substitution between e and $1 - r$ in equation (9) is $-1/(1 - \sigma)$.

For σ equal to zero, equation (9) becomes a Cobb-Douglas production technology, $c = F(e, r) = e^\alpha (1 - r)^\beta$. For σ equal to one, equation (9) becomes a linear production technology, $c = F(e, r) = \alpha e + \beta(1 - r)$. With a linear technology the heretofore unappropriated resource and time and effort, although distinct, are perfect substitutes in production.[9]

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[9]Another possible, albeit strained, interpretation of the special case of a linear production technology is

The economic problem in this model is that the appropriation of the heretofore unappropriated resource and the production of consumables are alternative uses of time and effort. Each agent must choose how to allocate its one unit of time and effort between these activities.

Equation (9) implies that equilibrium consumption, c^*, equals $[\alpha E^\sigma + \beta(1 - R^*)^\sigma]^{1/\sigma}$. Thus, c^* is a decreasing concave function of R^* for σ smaller than one and a decreasing linear function of R^* for σ equal to one. Potential consumption, $\hat{c}$, equals $(\alpha E^\sigma + \beta)^{1/\sigma}$.

Equation (9) also implies that $\partial F(E, R^*)/\partial e$ equals $\alpha(c^*/E)^{1-\sigma}$ and that $\partial F(E, R^*)/\partial r$ equals $-\beta[c^*/(1 - R^*)]^{1-\sigma}$. Hence, with two factors and a CES technology the equilibrium condition for determining R^*, equation (5), becomes

$$(10) \qquad m \, \frac{n}{n+1} \, \frac{E}{R^*} = \frac{\beta}{\alpha} \left(\frac{E}{1 - R^*} \right)^{1-\sigma}.$$

3.1. Cobb-Douglas Technology

To explore further the implications of allowing for two factors, we have to distinguish the case of CES technology with σ less than or equal to zero from the case of CES technology with σ positive, but not larger than one. Consider the case of $\sigma \leq 0$. In this case the constant elasticity of substitution is between zero and minus one. For simplicity, and without loss of generality, we can focus on σ equal to zero, the Cobb-Douglas technology.

With $\sigma \leq 0$, equation (9) implies that c goes to zero as r goes to one. In other words, with $\sigma \leq 0$, time and effort are essential for production. Accordingly, as in the one-factor model, for any positive value of $\underline{c}$, no matter how small, a large enough value of R^* would be inconsistent with the requirement that c^* be as large as $\underline{c}$. More precisely, with the Cobb-Douglas technology, for c^* to be as large as $\underline{c}$, R^* cannot be larger than $1 - (\underline{c}/\hat{c})^{1/\beta}$, where $\hat{c}$, potential consumption, equals E^α.

that there is only one resource, with each agent having an inalienable endowment of some of it and with the rest of it heretofore unappropriated, but that agents can use only their inalienable endowments in the appropriative competition.

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But, with $\sigma \leq 0$, equation (10) implies that R^* is not proportionate to m. This property obtains because, with $\sigma \leq 0$, e and r are not perfect substitutes in the production of consumables. Instead, the ratio of $-\partial F(E, R^*)/\partial r$ to $\partial F(E, R^*)/\partial e$ is an increasing convex function of R^* that goes to infinity as R^* approaches one. As a result, equation (10) implies that R^* is an increasing concave function of m, which only approaches one asymptotically as m goes to infinity.

Given this relation between R^* and m, c^* is a decreasing function of m, but c^* only approaches zero asymptotically as m goes to infinity. These results say that, because the ratio of $-\partial F(E, R^*)/\partial r$ to $\partial F(E, R^*)/\partial e$ goes to infinity as R^* approaches one, for any finite value of m agents allocate a positive fraction of their time and effort to production. In Figure 1 the convex locus that is asymptotic to the horizontal axis illustrates the relation between c^* and m for the two-factor model with a Cobb-Douglas technology.[10]

Let $\overline{m}_{CD}$ denote the value of $\overline{m}$ with a Cobb-Douglas technology. By the definition of $\overline{m}$, for m equal to $\overline{m}_{CD}$, c^* equals $\underline{c}$. By varying $\underline{c}$ in Figure 1, we see that $\overline{m}_{CD}$ is a decreasing convex function of $\underline{c}$. More precisely, using equation (10), with σ equal to zero, the condition for the viability of anarchy, $c^* \geq \underline{c}$, becomes $m \leq \overline{m}_{CD}$, where

$$(11) \qquad \overline{m}_{CD} \;=\; \left[\left(\frac{\hat{c}}{\underline{c}}\right)^{1/\beta} - 1\right] \frac{\beta}{\alpha} \frac{n+1}{n}.$$

According to equation (11), and as in the one-factor model, as $\underline{c}$ approaches $\hat{c}$, $\overline{m}$ approaches zero. Again, if adequate consumption is almost as large as potential consumption, then anarchy is unlikely to be viable.

But, equation (11) also implies that, as $\underline{c}$ approaches zero, $\overline{m}_{CD}$ increases without bound. This result says that a model with a Cobb-Douglas production technology, or,

[10]In drawing this figure we have normalized potential consumption, $\hat{c}$, to be the same in the two-factor model with a Cobb-Douglas technology as in the one factor model. For a Cobb-Douglas technology $\hat{c}$ equals E^α. Thus, this normalization implies that α equals γ. This normalization also implies that at the vertical axis the slope of the convex locus equals the slope of the concave locus derived from the one-factor model.

more generally, with a CES production technology with σ less than or equal to zero, is qualitatively different from the one-factor model. The important consequence of equation (11) is that, if adequate consumption is small, relative to potential consumption, then, with a Cobb-Douglas production technology, and, more generally, with a CES production technology with σ less than or equal to zero, only an extremely large decisiveness parameter would undermine the viability of anarchy.

3.2. Linear Technology

Now consider the case of CES technology with σ positive. In this case the constant elasticity of substitution is less than minus one. For simplicity, and without loss of generality, we can focus on σ equal to one, the linear technology in which the heretofore unappropriated resource and time and effort, although distinct, are perfect substitutes in production.

With $\sigma > 0$, equation (9) implies that time and effort are not essential for production, because with $\sigma > 0$, c goes to αe, and not to zero, as r goes to one. Accordingly, if $\underline{c}$ is not larger than αE, then anarchy is viable for any feasible value of R^*. Conversely, a large enough value of R^* would be inconsistent with the requirement that c^* be as large as $\underline{c}$ if and only if $\underline{c}$ is larger than αE. More precisely, with a linear production technology, for c^* to be as large as $\underline{c}$, R^* cannot be larger than $min\{(\hat{c} - \underline{c})/\beta, 1\}$, where $\hat{c}$, potential consumption, equals $\alpha E + \beta$.

With $1 > \sigma > 0$, equation (10) still implies that R^* is an increasing concave function of m, which only approaches one asymptotically as m goes to infinity. With $\sigma = 1$, equation (10) implies that R^* is an increasing linear function of m for $m < (1 + 1/n)\,(\beta/\alpha)\,(1/E)$, and that R^* equals one for $m \geq (1 + 1/n)\,(\beta/\alpha)\,(1/E)$.

Given these relations between R^* and m, with $1 > \sigma > 0$, c^* is a decreasing convex function of m, which approaches αE asymptotically as m goes to infinity. With $\sigma = 1$, c^* is a decreasing linear function of m for $m < (1 + 1/n)\,(\beta/\alpha)\,(1/E)$, and c^* equals αE for $m \geq (1 + 1/n)\,(\beta/\alpha)\,(1/E)$. These results say that, because with positive σ time

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and effort are not essential for production, consumption is positive for any finite value of m. In Figure 1 the piece-wise linear locus illustrates the relation between c^* and m for the two-factor model with a linear production technology.[11]

Let $\overline{m}_L$ denote the value of $\overline{m}$ with a linear technology. By the definition of $\overline{m}$, for m equal to $\overline{m}_L$, c^* equals $\underline{c}$. Using equation (10), with σ equal to one, the condition for the viability of anarchy, $c^* \geq \underline{c}$, becomes $m \leq \overline{m}_L$, where

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(12) \qquad \overline{m}_L \; = \; \begin{cases} \infty & \text{for} \quad \underline{c} \leq \alpha E \\[2em] \dfrac{\hat{c} - \underline{c}}{\alpha E} \dfrac{n+1}{n} & \text{for} \quad \underline{c} > \alpha E. \end{cases}
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According to equation (12), and as in the previous models, as $\underline{c}$ approaches $\hat{c}$, $\overline{m}$ approaches zero. Again, if adequate consumption is almost as large as potential consumption, then anarchy is unlikely to be viable.

But, equation (12) also implies that, if, as depicted in Figure 1, $\underline{c}$ is not larger than αE, then the requirement that c^* be as large as $\underline{c}$ is satisfied for all values of m. Thus, a model with a linear production technology, or, more generally, with a CES production technology with positive σ, also is qualitatively different from the one-factor model. With a linear production technology, and, more generally, with a CES production technology with positive σ, if adequate consumption is not too large, then even an extremely large decisiveness parameter would not undermine the viability of anarchy. Alternatively, if $\underline{c}$ is larger than αE, then the requirement that c^* be as large as $\underline{c}$ would not be satisfied for sufficiently large values of m. But, to undermine the viability of anarchy the decisiveness parameter again would have to be larger than in the one-factor model.

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[11]Because for a linear technology $\hat{c}$ equals $\alpha E + \beta$, the normalization that potential consumption is the same in the two-factor model with a linear technology as in the one factor model implies that $\alpha E + \beta$ equals E^{γ}. Also, we have drawn the slope of the linear locus as equal to the slopes of the other loci at the vertical axis. This construct implies that we also have normalized E to equal one.

4. Summary

This paper explored how the viability of anarchy depends on the decisiveness parameter that determines the marginal effect of allocating a resource to an appropriative competition. In a one-factor model in which agents use in the appropriative competition the same resource that they are competing to appropriate, anarchy appears to be fragile, because in this model equilibrium consumption would be adequate for the viability of anarchy only if the decisiveness parameter were small. After analyzing such a model, we generalized the analysis to allow the resource that agents are competing to appropriate to be distinct from the resource that they use in the appropriative competition.

Specifically, we assumed that the appropriative competition requires time and effort. We also assumed that an agent's consumption depends on the amount of the heretofore unappropriated resource that the agent appropriates and on the amount of time and effort that the agent allocates to production according to a standard constant-elasticity-of-substitution production technology. In this extended two-factor model the appropriative competition and the production of consumables are alternative uses of time and effort.

Our analysis revealed that in any model, if the amount of consumption that is adequate for the viability of anarchy is too large, then anarchy is not viable. But, our analysis also revealed that the one-factor assumption, which also presents a chicken-and-egg conundrum, is not an innocent simplification. Specifically, the implication that equilibrium consumption would be adequate for the viability of anarchy only if the decisiveness parameter were small is not robust.

In our extended two-factor model, we found that, with an elasticity of substitution between the heretofore unappropriated resource and time and effort between zero and minus one, if adequate consumption is not too large relative to potential consumption, then only an extremely large decisiveness parameter would undermine the viability of anarchy. Furthermore, we found that, with an elasticity of substitution less than minus one, if adequate

consumption is not too large relative to potential consumption, then even an extremely large decisiveness parameter would not undermine the viability of anarchy. In sum, we found that, with an apparently realistic distinction between the resource that agents compete to appropriate and the resource that agents use in the appropriative competition, anarchy is not fragile. Theory does not belie the observation that anarchy is prevalent.

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